Dynamics of a point mass. Newton's first principle. Inertial reference systems. Basic dynamic quantities – force, mass, momentum. Newton's second principle. Forces' addition – principle of superposition. Newton's third principle. Mechanical system. Law of change and conservation of momentum of a mechanical system of point masses. Types of forces in mechanics. Gravitational force. Forces of gravity, pressure and support reaction. Elastic force. Frictional force

Newton's first principle. Inertial reference system

Dynamics is a part of mechanics, in which its basic laws (principles) are formulated, determining the relationship between the forces acting on a body and its movement.

Principles represent such general laws that cannot be directly proven, but are arrived at as a result of long life experience. Their validity is confirmed by the experimental verification of their numerous consequences.

The basic principles of dynamics were formulated by the English physicist Isaac Newton in 1687 in his book Mathematical Principles of Natural Philosophy. They are summaries of numerous experiments and observations.

Newton's first principle states that any point mass (body) maintains its state of rest or uniform rectilinear motion until some external force causes it to be displaced from that state.

The general property of bodies to maintain their state of rest or rectilinear uniform motion in the absence of external influences is called inertia. Therefore, if a body moves without external influence, we say that it moves by inertia. This is why Newton's first principle is also known as the law of inertia.

It turns out that this principle does not hold for every frame of reference. Newton formulated it in relation to a system which he assumed to be in a state of absolute rest. All the further development of physics shows that such a system does not exist. Reference systems in which the principle of inertia is fulfilled are called inertial. It has been experimentally proven that Newton's principle is carried out with great accuracy in the so-called heliocentric inertial system. A point on the surface of the Sun is chosen as the center of this system, and its axes are directed to three distant stars, chosen so that the axes are mutually perpendicular.

All reference coordinate systems that move rectilinearly and uniformly relative to a given inertial reference frame are also inertial systems. It follows that if there is one inertial frame of reference, then there are infinitely many inertial frames.

Basic dynamic quantities – force, mass, momentum. Newton's second principle

From Newton's first principle it follows that in order to change the magnitude and direction of the velocity of a body, it must experience some external force. As a result, the body changes its speed i.e. gains acceleration. The accelerated motion of the body is a manifestation of some new quality, differing essentially from the state of rest or rectilinear uniform motion. The occurrence of acceleration is usually associated with the action of forces. It is accepted that any cause of change in the velocity of a body is called a force. Force is a vector quantity denoted by $\vec{F}$. Each force is associated with some effect on the given body.

Experience has shown that when the same body is acted upon by forces of different magnitudes, the accelerations it acquires are proportional to the magnitudes of the forces. However, it turns out that if the same force acts on different bodies, their speed of movement changes in a different way, i.e. they acquire different accelerations. Therefore, the result of the action of the force depends not only on the force itself, but also on some characteristic specific to each body. This characteristic is called mass and is a scalar physics quantity that is introduced in classical mechanics as a quantitative measure of the inertia of bodies. It is denoted by m, and its unit of measurement is kilogram [kg] (base unit in **SI)**. Experiments show that the greater the mass of a body, the less acceleration it receives under the action of a given force.

Newton's second principle defines the relationship between force, mass, and acceleration. It states that the acceleration experienced by a body is proportional to the force acting on it and inversely proportional to the mass of the body:

$$\vec{a} = k\frac{\vec{F}}{m}.$$

From the formula it can be concluded that the acceleration is always directed in the direction of the acting force. The proportionality factor depends on the chosen system of measurement units. In the **SI** system, the basic units of measurement are chosen so that the coefficient $k = 1$. Thus we arrive at our familiar expression of the second principle:

$$(1)\quad \vec{a} = \frac{\vec{F}}{m}.$$

By converting (1) we can obtain the unit of measurement for force, which in **SI** is named after Newton [N]:

(2) $\vec{F} = m\vec{a} \Rightarrow$ [N]=[kg.m/s ^{2}].

The formulations of the second principle in the form (1) and (2) are mathematically equivalent, and each of them can be used to solve specific problems. It should not be forgotten, however, that (1) is the correct formulation of the principle, since it takes into account the cause-and-effect relationship between phenomena - the application of force causes accelerated motion, not the other way around, i.e. acceleration is a function of applied force.

Another important quantity in mechanics is momentum of a body. It is defined as the product of the body's mass by its velocity. It is denoted by $\vec{p}$ and, as seen from the definition, is a vector quantity:

(3) $\vec{p} = m\vec{v}$.

The unit of measurement is [kg.m/s].

Momentum is one of the most important quantities not only in mechanics, but also in all of physics. The main reason is that it is one of the few quantities for which a universal conservation law can be formulated.

Newton's second principle can also be formulated in terms of momentum (this is also Newton's original formulation). If we take the first derivative of (3) with time and bearing in mind that the first derivative of the velocity is the acceleration, and the force can be expressed by (2), we get:

(4) $$\frac{d\vec{p}}{dt} = \frac{d\left(m\vec{v}\right)}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a} = \vec{F},$$

i.e. the applied force causes the body's momentum to change. This formulation is more general, as it is also valid in relativistic mechanics (for motion with velocities close to the speed of light), unlike (1).

Forces' addition – principle of superposition

If several forces act on a body, it receives acceleration in the direction of the vector sum of all acting forces, which is called equivalent:

(5) $$\vec{F} = \vec{F_1} + \vec{F_2} + \ldots + \vec{F_N} = \sum_{i=1}^{N} \vec{F_i}$$

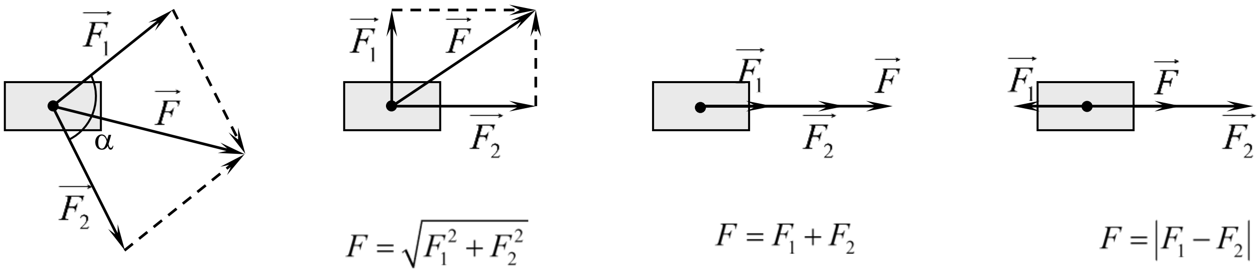
Using (5), we can generalize the basic dynamic equation (Newton's second principle) (1):

$$\vec{a} = \frac{\vec{F}}{m} = \frac{1}{m}\left(\vec{F_1} + \vec{F_2} + \ldots + \vec{F_N}\right) = \frac{1}{m}\sum_{i=1}^{N} \vec{F_i}.$$

We can also generalize the formulation by the momentum change (4):

$$\frac{d\vec{p}}{dt} = \sum_{i=1}^{N} \vec{F_i}.$$

Determining the equivalent of several forces as the vector sum of these forces (5) is an application of a general principle that applies to vector quantities – the principle of superposition (of vector addition). It is also called the principle of independent action when it applies to quantities such as force or intensity. Its essence in the specific case is expressed in the fact that when we consider the result of the action of one force, we are not interested in the other acting forces (independent action of each force), but the total result of all acting forces is obtained by vector addition (superposition) of the individual results. The principle of superposition also has the opposite effect - we can decompose a vector quantity (e.g. force, velocity, acceleration) into several vector components, because the principle guarantees us that the sum of their individual actions is equal to the action of their vector sum, i.e. to the corresponding unexpanded vector quantity.



a) b) c) d)

Fig. 1

The principle of superposition can be illustrated by several examples.

If a body moves under the action of the two forces $\vec{F_1}$ and $\vec{F_2}$ (Fig. 1a), the movement will be in the direction of the equivalent force $\vec{F}$, which is the vector sum of $\vec{F_1}$ and $\vec{F_2}$. Its magnitude is determined by the cosine theorem:

$$\vec{F} = \vec{F_1} + \vec{F_2}$$
$$F \equiv \left|\vec{F}\right| = \sqrt{F_1^2 + F_2^2 - 2F_1F_2\cos(\pi - \alpha)}.$$

When α= π/2 rad (Fig. 1b), the formula for the magnitude of the force $\vec{F}$ is slightly simplified (cos(π–α) = 0). And if the two forces act along a straight line (Fig. 1c, d), the magnitude of the equivalent force $\vec{F}$ is simply the sum (Fig. 1c) or difference (Fig. 1d) of the magnitudes of the two forces $\vec{F_1}$ and $\vec{F_2}$. It should always be kept in mind, however, that we only calculate the magnitude of the force in this way. The equivalent force itself $\vec{F}$ is always a vector sum (not a difference) of all acting forces (5), regardless of their direction. Equality (5) is the mathematical expression of the superposition principle for forces.

Very often, the principle of superposition is also used in the opposite direction – to decompose vector quantities into their components. If a body is thrown from some height *h* at an angle α to the Earth's surface with a velocity $\vec{v}$ (Fig. 2), it will move along a curved line (parabola) towards the Earth. However, we can represent this curvilinear movement as the sum of two straight line movements - one in the horizontal direction is uniform with the obtained initial velocity $\vec{v_x}$, because no force acts in this direction, and the other in the vertical direction is uniformly accelerated with acceleration $\vec{g}$ and initial velocity $\vec{v_y}$. This decomposition is possible because of the principle of superposition – the sums of the initial velocities and accelerations of the two rectilinear movements are equal to the initial velocity and acceleration of the curved one:

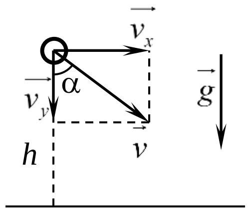


Fig. 2

$$\vec{v} = \vec{v_x} + \vec{v_y}$$
$$\vec{a} = \vec{g} + 0 = \vec{g}.$$

Newton's third principle. Mechanical system

So far, through Newton's first two principles, we have clarified what happens to a body when there is no or no force on it. The first principle considers an isolated body. In the second principle, another body (or bodies) is implicitly introduced, insofar as the external influence can only come from some other body. There, however, we were mainly concerned with the result of this impact - the change in the velocity of the body. On the other hand, it is logical to assume that if the body **A** we are considering is subjected to an impact from another body **B**, then the body **B** will also be subjected to some impact from the body **A**, i.e. bodies interact with some forces. Newton's third principle gives us the relationship between these forces.

Newton's third principle states that the forces with which two bodies interact are equal in magnitude and opposite in direction.

(6) $\vec{F_{12}} = -\vec{F_{21}}$

If body **1** acts on body **2** with a force $\vec{F_{12}}$ (Fig. 3), then the force $\vec{F_{21}}$ with which body **2** acts on body **1** is directed along the same line as $\vec{F_{12}}$, has the same magnitude, but is in the opposite direction. It should be taken into account, however, that these forces have different application points – the force $\vec{F_{21}}$ acts on body **1,** and the force $\vec{F_{12}}$ acts on body **2.** Therefore, these forces cannot balance each other.

Fig. 3

Newton's third principle, which introduces an explicit second body and defines interaction forces, now enables us to define interactions between any numbers of bodies. Therefore, it is necessary to introduce another concept - a mechanical system. It is a set of bodies between which mechanical forces act. The forces that act on the bodies of the system can generally be divided into two types - internal forces, which are caused only by interactions of the bodies of the system, and external forces, caused by the interaction of the bodies of the system with external bodies.

If only internal forces act in a mechanical system, it is called closed (isolated), and if external forces also act, it is called open.

Having defined a mechanical system and an interaction between more than two bodies, we can also introduce the magnitude of momentum of a system of bodies. The momentum of a mechanical system of bodies is defined in a similar way to the equivalent force - through the principle of superposition. The total momentum of the system is equal to the vector sum of the momenta of the individual bodies:

$$(7)\quad \vec{p} = \sum_{i=1}^{\mathrm{N}} \vec{p_i}\ .$$

Law of change and conservation of momentum of a mechanical system of point masses

We will use Newton's principles to prove one of the fundamental laws in physics - the law of conservation of momentum. Let us consider a mechanical system of **N** bodies (Fig. 4). (For clarity, Fig. 4 is made for only three bodies.) Two types of forces act on each of the bodies - internal forces between the bodies in the system, which we will denote by $\vec{f_{ij}}$ (the force with which body *i* acts on body *j* of the system) and external forces, which we will denote by $\vec{F_i}$ (the equivalent of all external forces acting on body *i*). If we write down the basic dynamic equation expressed in terms of the change in momentum for each of the bodies in the mechanical system, we get the following system of equations:

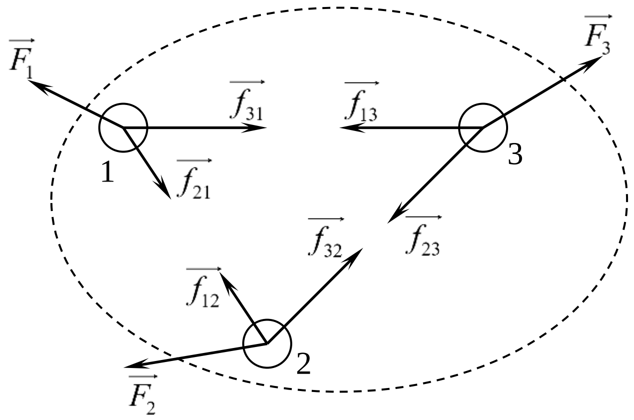


Fig. 4

$$\frac{d\vec{p_1}}{dt} = \vec{f_{21}} + \vec{f_{31}} + \ldots + \vec{f_{\mathrm{N}1}} + \vec{F_1},$$

$$\frac{d\vec{p_2}}{dt} = \vec{f_{12}} + \vec{f_{32}} + \ldots + \vec{f_{\mathrm{N}2}} + \vec{F_2},$$

$$(8)\quad \frac{d\vec{p_3}}{dt} = \vec{f_{13}} + \vec{f_{23}} + \ldots + \vec{f_{\mathrm{N}3}} + \vec{F_3},$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\frac{d\vec{p_\mathrm{N}}}{dt} = \vec{f_{1\mathrm{N}}} + \vec{f_{2\mathrm{N}}} + \ldots + \vec{f_{(\mathrm{N}-1)\mathrm{N}}} + \vec{F_\mathrm{N}}.$$

In this system of equations (8) e.g. $\frac{d\vec{p_1}}{dt}$ is the change in momentum of body **1** for the infinitesimally small time interval *dt*, $\vec{f_{21}}$ is the force with which body **2** acts on body **1**, $\vec{F_1}$ is the equivalent of all external forces acting on body **1**, etc.

If we collect all the equations of the system (8), on the left side we will get the sum of the derivatives of the momenta of all bodies over time (the changes of all momenta for the infinitesimally small time interval *dt*):

$$(9)\quad \frac{d\vec{p_1}}{dt} + \frac{d\vec{p_2}}{dt} + \frac{d\vec{p_3}}{dt} + \ldots + \frac{d\vec{p_\mathrm{N}}}{\mathrm{dt}} = \frac{d}{dt}\sum_{i=1}^{\mathrm{N}} \vec{p_i} = \frac{d\vec{p}}{dt}$$

and using the sum derivative rule and (7) we see that this is the change in momentum of the mechanical system for the time interval *dt* .

On the right side of the equation, we will get the sum of all the forces that act on all the bodies in the system:

$$(10)\quad \vec{f_{21}} + \vec{f_{12}} + \vec{f_{31}} + \vec{f_{13}} + \vec{f_{\mathrm{N}1}} + \vec{f_{1\mathrm{N}}} + \vec{f_{32}} + \vec{f_{23}} + \ldots + \vec{f_{\mathrm{N}(\mathrm{N}-1)}} + \vec{f_{(\mathrm{N}-1)\mathrm{N}}} + \vec{F_1} + \vec{F_2} + \vec{F_3} + \ldots + \vec{F_\mathrm{N}} = \sum_{i=1}^{\mathrm{N}} \vec{F_i}\ ,$$

since according to (1) the sum of the internal forces is equal to zero ($\vec{f_{21}} = -\vec{f_{12}};\ \vec{f_{31}} = -\vec{f_{13}}$ etc.). Since (9) must be equal to (10), we finally get:

$$(11)\quad \frac{d\vec{p}}{dt} = \sum_{i=1}^{\mathrm{N}} \vec{F_i}\ ,$$

i.e. the change in momentum of a mechanical system per unit time is equal to the equivalent of all external forces acting on all bodies of the system. We performed the calculation for a random system. If the mechanical system is closed (no external forces acting), then the right-hand side of (11) will be equal to zero. This means that the quantity $\vec{p}$ does not change with time (its first derivative is zero, so the quantity is a constant). But $\vec{p}$ it is the full momentum of the mechanical system. This is how we arrive at the law of conservation of momentum in a closed mechanical system:

$$\frac{d\vec{p}}{dt} = 0 \text{ or } \vec{p} = \text{const}.$$

The total momentum of a closed mechanical system does not change with time.

Jellyfish, rocket, walking...

If the system is open, then the change in momentum of a mechanical system can only be caused by external forces (11). Forces internal to the system can only cause a redistribution of momentum between the bodies of the system.

Types of forces in mechanics. Gravitational force

When describing mechanical phenomena, we will use different forces - gravity, pressure, friction, etc. On the other hand, we know that there are only four types of interactions in nature and the corresponding forces associated with them. An interesting question is how these four fundamental forces are related to the diverse forces manifesting in the reality that surrounds us. It turns out that the forces in mechanics are most often the result (equivalent) of a large number of fundamental forces (mainly gravitational and electromagnetic) applied at different points of the considered body. The main difference between the fundamental and other forces is the simplicity of the laws that describe the fundamental forces. For example, the gravitational force of interaction between two stationary material points with masses m_1 and m_2, located at a distance r from each other, is given by Newton's law of universal attraction:

$$F_g = \gamma \frac{m_1 m_2}{r^2},$$

and the electric force of interaction between two stationary point charges - from Coulomb's law, which has the same form, but the masses are replaced by the charges q_1 and q_2:

$$F_e = k \frac{q_1 q_2}{r^2}.$$

In contrast to these simple expressions, the dependences that define the forces we observe are in most cases much more complex, and sometimes (for example, for frictional forces) they can only be obtained empirically, i.e. based on experience. The reason is that these forces depend on a large number of fundamental interactions, sometimes of a different kind, which is why it is impossible to find an exact mathematical formula for their description. For example, the force of gravity of a body depends only on the gravitational interaction of the material points of which it is built with the Earth, and therefore the formula is easily obtained. The same applies to a charged body placed in an electrostatic field - there the interaction is only electrostatic. But the frictional forces are obtained as the equivalent of many gravitational forces of interaction of the points of the body with the Earth (which determine the weight of the body) and many electromagnetic forces between the atoms and molecules of the body and the substrate (which determine the coefficient of friction). Therefore, the laws determining the forces of friction are only empirical.

Force of gravity, pressure and support reaction

An experimental fact (Galilei's experiments) is that all bodies fall (freely) towards the Earth with the same acceleration (if we ignore the rotation of the Earth on its axis). Therefore, any object near the Earth must be acted upon by a force that depends only on its mass. This force is caused by the gravitational attraction between the Earth and the body and is called the force of gravity $\vec{G}$. It is analytically expressed by the formula:

$$\vec{G} = m\vec{g},$$

where ***m*** is the mass of the body, and $\vec{g}$ - the constant acceleration with which all bodies fall freely, called terrestrial acceleration. It is important to note that the application point of $\vec{G}$ is in the body itself, and its direction is towards the center of the Earth. If the body is not free (e.g. placed on a support (Fig. 5) or suspended by a thread) it will not move, even though gravity acts on it. Therefore, another force $\vec{R}$ must act on the body, which we call the reaction of the support (or thread tension, in this case it is most often marked with $\vec{T}$). It is also applied in the body (otherwise it will not balance $\vec{G}$), not in the support. According to Newton's third principle, when a support acts on a body, the body must also act on the support with

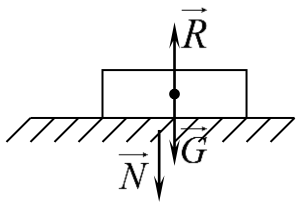


Fig. 5

a force equal in magnitude and opposite in direction. This force is called pressure $\vec{N}$ (very often also called weight). Unlike the other two forces considered, it is applied at the support.

The force of gravity and weight (pressure) are often confused. From the definitions of the two forces, it can be seen that for the force of gravity to act, it is enough for the body to be close to the Earth. The force of gravity $\vec{G}$ has nothing to do with the interaction of a body with other bodies. The force $\vec{N}$, however, is directly related to the support – if in fig. 5 we remove the support, the force $\vec{N}$ will no longer act – the body will be in a state of weightlessness until $\vec{G}$ does not change. We will get the same result if the body and support fall freely together – the body will not act on the support. The mistake of confusing the two forces comes from the fact that in the most common case (the support is horizontal and the body and support are stationary) the two forces are equal in magnitude and directed in the same direction. It should not be forgotten, however, that they have different application points – $\vec{G}$ is applied in the body, and $\vec{N}$ – in the support or thread.

Weightlessness, satellite...

Forces of elasticity and friction

All real bodies under the action of forces deform, i.e. change their size and shape. If, after the termination of the action of the external force, the body recovers its size and shape, the deformation is called elastic. Let us consider the deformation of a spring on which we act with a force $\vec{F}$ along the length of the spring (Fig. 6a).

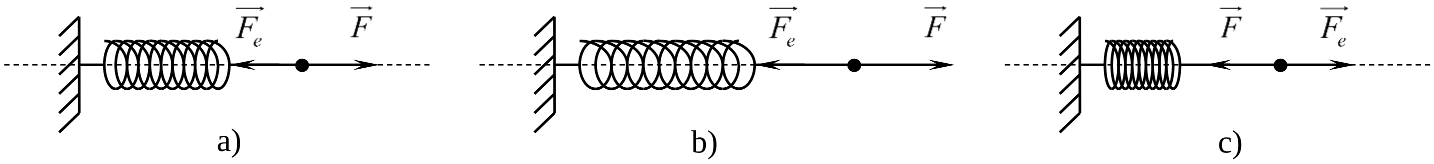


Fig. 6

We assume that the force is small enough to stay within the bounds of elastic deformation. The spring is in equilibrium, so this force $\vec{F}$ must be balanced by some other force. This force is called the elastic force $\vec{F}_e$ of the spring and is equal in magnitude to the applied force $\vec{F}$ and opposite in direction. If we apply a greater force (Fig. 6b), the spring also remains in equilibrium, but we see that it has stretched more, i.e. the elastic strength has increased. If we apply the force $\vec{F}$ in the opposite direction (compress the spring, Fig. 6c), the elastic force $\vec{F}_e$ also changes its direction and the spring is again in equilibrium. Based on such experiments, the empirical law for the magnitude and direction of the elastic force was established:

(12) $\vec{F}_e = -k\vec{\Delta x}$,

where k is a proportionality factor depending on the characteristics of the spring itself and its dimension, as seen from (12), is N/m, and $|\vec{\Delta x}|$ is the change in length (lengthening or shortening) of the spring relative to its initial length x_0. The direction of $\vec{\Delta x}$ is in the direction of the external force $\vec{F}$ (Fig. 7). Since the forces act along only one axis, (12) is also written without vectors:

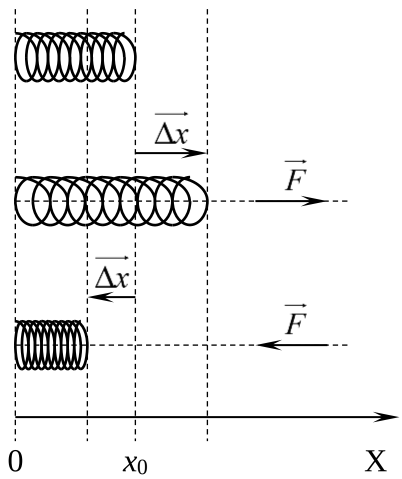


Fig. 7

(13) $F_e = -k\Delta x$,

but it should not be forgotten that the minus sign in (13) simply indicates that the elastic force and elongation have opposite directions, and is not related to the magnitude of the elastic force (the magnitude of the force cannot be negative).

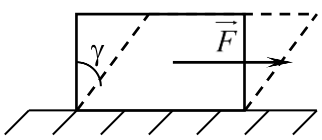


Fig. 8

Elastic forces arise not only in springs, but also in all bodies subjected to deformation. In addition to elastic forces during stretching and compression, such forces also occur during bending, twisting, and sliding. In all cases the force is determined by a formula similar to (13), the coefficient $\boldsymbol{k}$ being different and the change in length $\boldsymbol{\Delta x}$ may be replaced by another variable, e.g. angle $\boldsymbol{\gamma}$ under the elastic sliding forces (Fig. 8) or body twist angle, in the case of torsional deformation.

Another type of forces whose magnitude is determined only empirically are frictional forces. Let a body rest on a horizontal support (Fig. 9). If we apply a horizontal force $\vec{F}$, the body remains at rest. From the basic dynamic equation (Newton's second principle) it follows that the body must be acted upon by some other force that balances the applied force $\vec{F}$. It must have the same magnitude and opposite direction. We call this force friction force (dry friction force) at rest $\vec{F}_s$. If we apply a greater external force, the body also remains at rest - the force of friction at rest has become greater. When the magnitude of the applied force $\vec{F}$ exceeds some critical value F_0, the body begins to move. We call this value the maximum frictional force at rest (or just the frictional force at rest). It was experimentally established that the magnitude F_0 of this force does not depend on the area of the body and the support, but only on the magnitude of the normal pressure $\vec{N}$ (weight) of the body (and not on the force of gravity $\vec{G}$!) and the material from which the body and the pad. This dependence can be expressed by an empirical law similar to (13):

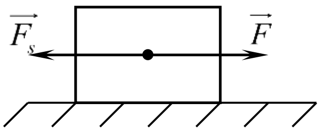


Fig. 9

(14) $F_0 = k_0 N$.

Glass...

The coefficient k_0 depends on the material from which the body and the pad are made and on the condition of their surfaces and is called the coefficient of friction at rest. It is determined experimentally for each pair of substances. It can be seen from (14) that k_0 is a dimensionless quantity (number).

When the body begins to move (slide) on the pad, it also experiences a frictional force called the sliding frictional force. Its size is determined in the same way:

$F_s = kN$,

but the coefficient k is different from k_0. It is also determined experimentally for each pair of substances, but also depends on the speed of movement. Therefore, the force of friction during sliding depends on the speed. This dependence is shown in Fig. 10. We see that the sliding friction force slightly decreases at low speed (compared to the maximum value of the friction force at rest F_0), then begins to increase slightly. However, this variation is very small, so at low speeds it is usually considered that $k \approx k_0$=const.

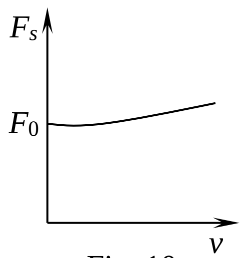


Fig. 10

Helpful or harmful...

In addition to the forces of friction at rest and sliding, there is also the force of rolling friction. It is much smaller than the sliding friction force and also depends on the radius of the rolling body. In cases where the frictional force is harmful and must be reduced, a way is sought to replace sliding with rolling (e.g. in bearings). Where this is not possible, some non-wetting fluid (e.g. machine oil) is placed between the rubbing parts, because then the friction (called wet friction) is significantly less than dry friction.

Grease?

The movement of bodies around the Earth is strongly influenced by the force of air resistance.

Parachute